## ADVANCED BUSINESS MATHEMATICS - HONOURS

## Sixth Paper

Full Marks : 50
The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

## Group - A

1. Answer any four questions:
(a) Find the domain of definition of the function $\log \left(x^{2}-5 x+6\right)$

## Or,

If $f(x)=[x-1]-x$, then find $f(0)$ and $f(1)$.
(b) Draw the graph of the function $f(x)=|x|$.
(c) If $f(x)=\frac{x-1}{x+1}$, then show that $\frac{f(a)-f(b)}{1+f(a) f(b)}=\frac{a-b}{1+a b}$

Or,
If $3 f(x)-2 f(-x)=10 x-1$, find $f(x)$ and hence find $f(2-3 x)$.
(d) Find $\lim _{x \rightarrow-4}\left[\frac{1}{x+4}+\frac{8}{x^{2}-16}\right]$
(e) Differentiate $e^{\sqrt{x}}$ with respect to $x$.

Or,
If $x^{2}+y^{2}=2 \log (x+y)$, find $\frac{d y}{d x}$.

## Group - B

2. Answer any two questions:
(a) Evaluate :
(i) $\lim _{x \rightarrow 0} \frac{e^{a x}-e^{b x}+k x}{x}$
(ii) $\lim _{x \rightarrow \infty} \frac{4 x^{4}-3 x+2}{5 x^{4}+2 x^{2}+3}$
(b) If $x=\frac{1-t}{1+t}$ and $y=\frac{2 t}{1+t}$, then show that $\frac{d^{2} y}{d x^{2}}=0$
(c) Show that the maximum value of $x+\frac{1}{x}$ is less than its minimum value.

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\boldsymbol{O r}
$$

The sum of two positive numbers is 24 . Find the two numbers if the sum of their squares is minimum.
(d) Prove that (without direct expansion)

$$
\begin{gathered}
\left|\begin{array}{lll}
x & a & b \\
a & x & b \\
a & b & x
\end{array}\right|=(x-a)(x-b)(x+a+b) \\
\text { Or, }
\end{gathered}
$$

Solve by matrix inversion method:

$$
x+y+z=3 ; x-y+z=1 ; x+y-z=1
$$

(e) If $A=\left[\begin{array}{rrr}2 & -1 & 1 \\ -2 & 3 & -2 \\ -4 & 4 & -3\end{array}\right]$, find $A^{2}-A$.

## Or,

Express the following in a single matrix :

$$
\left[\begin{array}{rrr}
3 & 2 & 5 \\
2 & -4 & 0
\end{array}\right] \cdot\left[\begin{array}{rr}
1 & 2 \\
2 & -1 \\
0 & 5
\end{array}\right]-\left[\begin{array}{rr}
7 & 29 \\
-6 & 8
\end{array}\right]
$$

## Group - C

3. Answer any one question :
(a) Evaluate
(i) $\int \frac{2 x+3}{3 x+2} d x$
(ii) $\int \frac{2 x d x}{2+x-x^{2}}$
(b) A number is chosen at random from the first 50 positive integers. Find the probability that the chosen number is divisible by 3 or 5 .
(c) If 10 consecutive days are chosen at random, what is the probability that two of them will be Saturdays?
